O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Spring 2008
Midterm Exam #1

Do All Five Problems

Name : ______________________________

E-Mail Address:_______________________________________

Problem 1:

A system has zero-state response

$$y(t) = f\{u(t)\} = \int_{-\infty}^{\infty} (\tau - t)^3 u(\tau) 1(t - \tau - 1) d\tau$$

where the unit-step function $1(\lambda)$ is defined by

$$1(\lambda - a) = \begin{cases} 0, & \text{for } \lambda < a \\ 1, & \text{for } \lambda \geq \text{a} \end{cases}.$$

Determine whether this system is or is not a) causal, b) time-varying, c) zero-memory and d) zero-state linear. Please justify your answer.

Problem 2:

Suppose we have a state-space realization given by A, b, c with the three chosen state variables $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$. Suppose we are now interested in the state variables $z = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}^T$, where $z_1 = k_1 x_1$, $z_2 = k_2 x_2$, and $z_3 = k_3 x_3$, and we let $\dot{z} = Fz + gu,\ y = hz$.

a) Write out the matrices F, g, h in terms of the elements of A, b, c and the scale factors k_1, k_2, k_3.

b) Suppose we wish to change the time scale and substitute $\tau = a_0 t$ into the equations. Repeat part a), showing how F, g, h depend on the time scale factor a_0 and the elements of A, b, c.

<u>Problem 3</u>:
Find an *observable* canonical form realization (in minimal order) from SISO continuous-time system given below:

$$t^2\ddot{y}(t)+(t-1)\dot{y}(t)+e^{-2t}y(t)=4\ddot{u}(t)+3t\dot{u}(t)-t^2u(t)\,.$$

Notice that gain blocks may be *time* dependent. Show the state space representation and its corresponding simulation diagram.

Problem 4:

Find a minimal *observable* canonical form realization (i.e., its simulation diagram and state space representation) for the following MISO system described by

$$H(s) = \begin{bmatrix} \dfrac{2s+3}{s^3+4s^2+5s+2} & \dfrac{s^2+2s+2}{s^4+3s^3+3s^2+s} \end{bmatrix}$$

Problem 5:
If $\{A, B, C, D\}$, D nonsingular, is a realization with $H(s) = C(sI - A)^{-1}B + D$, show that $\{A - BD^{-1}C, BD^{-1}, -D^{-1}C, D^{-1}\}$ is a realization for a multi-input multi-output system with transfer function $1/H(s)$.